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COMPTON SAILING AND STRONG POLARIZATION

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ABSTRACT

It is noted that a surface layer of matter in contact with a sufficiently super-Eddington, radially combed photon flux typically attains a relativistic coasting state whereby the radiation does not accelerate the matter. The final bulk Lorentz factor of this matter is therefore determined by geometry. Radiation that scatters off this layer is most likely to be observed along the velocity vector of the matter, where it would be most strongly polarized.

Subject headings: black hole physics — gamma-rays: bursts and theory

A remarkable feature of cosmologically distant gamma ray bursts is their high isotropic equivalent luminosities, which can exceed the Eddington limit by up to 14 orders of magnitude. Such a flux would sharply accelerate baryonic matter that stood in its way and, even for a small quantity of such matter, most of the GRB energy would be expended on its acceleration. This would require salvage mechanisms such as internal shocks, which, in order to be effective in converting efficiently to gamma radiation, must occur near or beyond the photosphere, i.e. at large radii.

Is it necessary to have any baryonic matter at all in GRB? If GRB emerge along field lines that thread an event horizon, they need not receive any baryons, and most of the pairs could annihilate at relatively small radii when the temperature falls below 2×10^8K. The indirect evidence for matter in the fireballs is a) non-thermal spectra, which suggest that much of the energy passes through charged particles at low optical depth, and b) the existence of afterglows, which would not be generated purely by gamma rays. There are, however, models that could yield non-thermal emission and afterglow without a significant amount of baryons. For example, fireball energy is easily converted to ultrahigh-energy particles if neutrons decay within the baryon-poor jet after creeping in across field lines (Eichler & Levinson 1999, Levinson & Eichler 2003). The ultra-high energy particles could carry enough energy per electron that they could deliver the required non-thermal gamma ray energy and afterglow energy while remaining optically thin to the non-thermal gamma rays. Other models posit pair creation purely from gamma rays (Thompson & Madau 2000, Beloborodov 2001) or Poynting flux (Lyutikov, Parlev, & Blandford 2003) at large radii. The photosphere, which would be at large radius for a baryon dominated plasma, can in view of these non-baryonic alternatives be at much smaller radius. To quantify this, note that if the energy that remains kinetic - ultimately destined to power the afterglow - is carried as normal electron-ion plasma, then the photosphere occurs at $r = 1/(1-\beta)\sigma_T n_e \sim 10^{12} L_{b50} (\frac{\Gamma}{10^2})^{-3} (\frac{\theta}{0.2})^{-2}$ cm. Here L_{b50} is the kinetic luminosity of the baryons, $4\pi\Gamma\gamma_p m_p n_e c^3 \theta^2 r^2$, in units of 10^{50}erg/s, the neutron to proton ratio in the ions is assumed to be unity, and we have tentatively included the possibility that the internal Lorentz factor γ_p for the ions may be larger than unity. These considerations would together suggest that $r_{12} \geq 1$ and that $\Gamma \geq 10^2$, which is widely though not universally believed for GRB fireballs. If, on the other hand, the fireball plasma is mostly pairs then the photosphere can be dictated by pair recombination and occur at considerably smaller radius, ($r_{ph} \overset{\sim}{<} 10^{11}$cm), e.g. Eichler (1994), Eichler & Levinson (2000).

A now familiar suggestion for getting kinetic fireball energy into gamma radiation, is via internal shocks near (Eichler 1994) or downstream of (Rees & Mezsaros 1994) the gamma-ray photosphere. The question still remains whether the soft gamma ray emission, is emitted from optically thin regions or from a photosphere, or both. The low frequency parts of GRB spectra are frequently too steeply rising, it is reported (e.g. Preece et al. 2002), to be consistent with optically thin, efficient synchrotron radiation. Alternative pictures, which might allow a steeper rise in the soft part of the spectrum, involve Compton upscattering (Shaviv & Dar 1995, Lazzati et al. 2000) of softer photons by the energy-bearing, relativistic outflow. Here too, however, the bulk and internal Lorentz factors must be fine tuned to get the right peak energy.

However, there is another alternative: that radiation from or near a photosphere is the primary energy source, this photosphere is largely determined by net annihilation of positrons - which which loosely associates the peak energy with $m_e c^2$ - and baryon involvement, while generating additional effects, occurs too far downstream to spoil this association. Recent evidence of GRB collimation suggests that the fireball, though sufficiently baryon-pure to allow high bulk Lorentz factors, is surrounded by baryon-rich material either the envelope of a host star, or a wind from the accretion disk (Levinson & Eichler 2000). This allows an entirely different set of models for GRB's, one in which the GRB fireball would be pure photons (or Poynting flux) except for matter that is "shaved off" the inner wall of the baryonic sheath that collimates the fireball. The shavings could be accelerated to relativistic Lorentz factors, and could play whatever role is ascribed to matter in GRB fireballs, such as eventually supplying non-thermal particles and generating afterglow. However, they need not be the primary source of soft gamma rays and they need not be subject to constraints on Γ derived from time variability.

The purpose of this Letter is to consider the interaction between a radiative fireball and a sheath of baryonic matter when the latter collimates the former. It is argued that the moving wall of matter that is in contact with the fireball maintains a Lorentz factor Γ that is the inverse of the sine of the impact angle between the photons and the outflow. This

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is due to the fact that this matter "sails" on the radiation pressure. This picture could yield an estimate of the Lorentz factor of the matter that is associated with GRB fireballs. It also predicts that the polarization is typically high; it can be close to 100 percent, and can exceed 60 percent with significant probability. This is considerably higher on average than in previous suggestions (e.g. Shaviv & Dar 1995, Eichler & Levinson 2003).

Compton equilibrium between matter and radiation pressure has previously been considered in the context of an accretion disk by Sikora and Wilson (1981) Phinney (1982), Sikora et al (1996). Begelman and Sikora (1987) considered polarization from scattering off outflow, but did not specifically consider the correlation of such polarization with Compton equilibrium. Beloborodov (1998) considered the polarization vector in the case of a 1-D outflow from a disk with Compton equilibrium at modest β, $\beta \sim 0.5$. The polarization he found is along the outflow axis, orthogonal to what is expected in the context of the present paper. The polarization effect proposed here is a consequence of high Lorentz factor combined with Compton equilibrium.

Consider a flux of $\frac{lL_{Edd}}{4\pi r^2}$, where $l \sim 10^{14}$. Suppose an electron ion pair is injected at radius $R_o = x_o R_{Sch}$, where R_{Sch} is the Schwarzchild radius. As shown below, the electron proton pair is accelerated radially to an asymptotic Γ of $(l/x_o)^{1/3}$ when $\Gamma \gg 1$. The fractional power 1/3 is due to the redshift suffered by the radiation in the frame of the accelerated matter. Note that for $l \sim 10^{14}$ and $x \sim 10^7$, the asymptotic Γ is of order 200, quite consistent with popular estimates.

At a collimating wall, however, the baryonic outflow can be constrained to be *non-radial*, and the radiation pressure can be *more* effective because there is less redshift. Though somewhat counterintuitive, this principle is similar to the fact that a ship sails faster when kept oblique to the wind by its rudder than when traveling directly downwind. If a collimating wall keeps the flow moving oblique to the radial direction (which we are assuming here is about the same as the direction of the photon wind) then the acceleration or deceleration can be far more effective. The extremely large super-Eddington factor l implies that the final Lorentz factor is determined by the collimation geometry.

The radiation force on an electron proton pair along its direction of motion $\hat{\beta}$, which moves at angle χ relative to the radial direction $\hat{r}$, is most easily calculated in the frame of the pair, where the cross section is the Thompson cross-section. In this frame the component of force $\mathbf{F}'$ along the direction of flow is given by

$$\mathbf{F}' \cdot \hat{\beta} = \sigma_\mathbf{T} \mathcal{F}' \cdot \hat{\beta}/\mathbf{c} \equiv \sigma_\mathbf{T} \int \mathbf{d^2\hat{k}'} \int_\mathbf{0}^\infty \mathbf{I}'(\nu', \mathbf{\hat{k}'})(\mathbf{\hat{k}'} \cdot \hat{\beta}) \mathbf{d}\nu'/\mathbf{c} \tag{1}$$

where $\mathcal{F}'$ is the comoving radiative flux, σ_T is the photon scattering cross section which throughout is assumed to be the Thomson scattering cross section. The acceleration along the wall in this frame, a', is $a' = \mathbf{F}' \cdot \hat{\beta}/\mathbf{m}$. In the limit of strong radial combing of the radiation, $I'(\nu', \hat{k}')$ can be factored into $\mathcal{F}'_\nu(\nu')\delta^2(\hat{k}' - \hat{r}')$. Using the invariance of $\int_0^\infty I'(\nu')d\nu'/\nu'^4$, we can write

$$\mathcal{F}'_\nu = \mathcal{F}_\nu \frac{\delta^2(\hat{k} - \hat{r})}{\delta^2(\hat{k}' - \hat{r}')} \frac{\nu'^4}{\nu^4} \tag{2}$$

where $\mathbf{\hat{r}}'$ is the k vector of the radially combed photons in the primed frame and $\int_0^\infty \mathcal{F}_\nu d\nu = lL_{edd}/4\pi r^2$. In the lab frame, $d\beta\Gamma/dt = \Gamma^3 a = a'$. The primed variables can be expressed in terms of the unprimed variables as follows: Using the aberration formula,

$$cos\chi' = [cos\chi - \beta]/[1 - \beta cos\chi] \tag{3}$$

and

$$\nu' = \nu\Gamma(1 - \beta cos\chi), \tag{4}$$

we can write

$$d\Gamma\beta/dt = \frac{lL_{Edd}\sigma_T}{4\pi r^2 m_p} \frac{\delta^2(\hat{k} - \hat{r})}{c\delta^2(\hat{k}' - \hat{r}')} \Gamma^4 (1 - \beta cos\chi)^4 \frac{(cos\chi - \beta)}{(1 - \beta cos\chi)}. \tag{5}$$

When $\Gamma \gg 1$ and $1 - cos\chi \ll 1 - \beta$, the quantity $cos\chi' = (1 - \beta cos\chi)^{-1}(cos\chi - \beta) \sim 1$, and $\delta^2(\hat{k} - \hat{r})/\delta^2(\hat{k}' - \hat{r}') \propto [d(1 - \hat{k}' \cdot \hat{r}']/[d(1 - \hat{k} \cdot \hat{r}] = (1 + \beta)/(1 - \beta)$, so

$$d\Gamma\beta/dt \sim d\Gamma/dt = -(4\Gamma)^{-2} l d(1/x) \tag{6}$$

and the solution as a function of radius xR_{Sch} is

$$\Gamma(x) = [\frac{3l}{4}(1/x_o - 1/x) + \Gamma_o^3]^{1/3}. \tag{7}$$

This solution could be applicable to loose material that suddenly finds itself in the path of the fireball, such as a decaying pickup neutron that has drifted in from the walls. For $x_o \sim 10^6$, corresponding to a radius of $\sim 10^{12}$cm, and $l \sim 10^{13}$ to 10^{14}, this yields an asymptotic Γ of several hundred, which is an acceptable value, though Poynting pressure could drive it to an even higher value. Also, the Eddington luminosity increases with the number of pairs per baryon, so l could effectively be higher than 10^{14}.

However, for $l/x \gg \Gamma^3$, Γ grows very rapidly, possibly until the condition $1 - cos\chi \ll 1 - \beta$ is no longer satisfied. If the acceleration were to lead to $1 - cos\chi \gg 1 - \beta$, it would follow that $1 - \beta cos\chi \sim 1 - cos\chi$, and the deceleration would then be extremely powerful. For $l/x \gg 1$, the value of Γ is rapidly reduced to the value where $cos\chi = \beta$, i.e. $\Gamma = 1/sin\chi$.

It is of course also possible that the condition $1 - cos\chi \ll 1 - \beta$ is never satisfied, e.g. β could be small and $1 - cos\chi$ could be of order unity. In this case the acceleration along the wall is even larger than given by equation (6) because

the radiation is not redshifted nearly as much; the Doppler factor $\Gamma(1-\beta cos\chi)$ that would have lowered the flux in the fluid frame when $1-cos\chi$ is negligible is now less devastating. This only strengthens the conclusion that the material is accelerated until (and only until) $\beta = cos\chi$.

The final value for Γ is thus $1/sin\chi$. By the same argument, thermal motion is strongly Compton cooled. If the source has a finite size, then the incident photons have some finite angular spread in the frame of the fluid. However, any Compton heating this may cause is modest, because the frequency in the fluid frame ν' is only $\nu/\Gamma \ll m_e c^2$. Moreover, if the cylindrical radius of the fluid r is much larger than the source size r_s, as we have assumed here, then the angular spread of the photons in the fluid frame can be shown to be small, of order r_s/r, (though larger than the angular extent of the source as seen from the point of photon-fluid impact). A more general treatment of finite source size will appear in a separate paper (Levinson and Eichler 2004).

The incident radiation thus makes an angle of $\pi/2$ relative to the direction of motion, as would wind relative to the motion of a frictionless sailboat whose motion is kept by its rudder oblique to the wind direction. For the extremely high super-Eddington factor l that is typical of GRB's, we believe that viscous drag by the inner layers does not change this conclusion significantly when the mean free scattering time of an ion is longer than the acceleration time to $\beta = cos\chi$.

The radiation, when scattered off the matter, is beamed most strongly in the direction of motion, which corresponds to 90 degree scattering, and thus full polarization along the direction $\hat{n} \times \beta$. This should be contrasted with the situation corresponding to the "head-on" approximation as termed by Begelman and Sikora (1987) in which the radiation in the frame of the scatterer is moving almost parallel to the lab frame velocity as seen by the scatterer. The head-on approximation would be natural when the flow is ultra-relativistic and the angle of incidence is physically unrelated to Γ. In this circumstance, beaming along the direction of motion gives no polarization, and it is only the finite spread of the beamed radiation that allows polarized contributions from scatterers moving slightly off the line of sight. For a hollow cone geometry, the maximum intensity is seen by observers with lines of sight on the cone. The maximum polarization they may observe is 25 percent. The polarization vector is perpendicular to the cone, i.e. each fluid element with direction $\hat{\beta}$ contributes polarization in the direction $(\hat{n}-\hat{\beta}) \times \hat{n}$. Sufficiently far away from the cone, i.e. sufficiently far into its interior or exterior, the polarization can be larger than 25 percent, but only at the expense of lowered intensity. Thus the V_{max} associated with large polarizations is relatively small and the probability of viewing polarization more than 75 percent, say, is only 0.2 (Eichler and Levinson 2003).

By contrast, scattering from a Compton sail reverses the roles of high and low polarization, so the largest V_{max} is associated with high polarization. In the limit of a line of sight of $\delta\theta$ from the edge of an infinitely thin cone, where $\delta\theta \ll \theta_0$, and $1/\Gamma \ll \theta_0$, most of the contribution to the line of sight comes from fluid elements with β near $\hat{n}$, i.e. $\beta - \hat{n} \sim \delta\theta$.

To set up a formal calculation, consider scattering off a thin cone of opening angle θ_o defined by the azimuthal angle ϕ running from 0 to 2π, and the line of sight is offset from the cone by angle Δ. Let us use coordinates such that the unit line of sight vector $\hat{n} = [sin(\theta_0+\Delta), 0, cos(\theta_0+\Delta)]$ is in x,z plane. The ingoing photon vector is $\vec{k}_1 = [sin(\theta_0+\frac{1}{\Gamma})cos\phi, sin(\theta_0+\frac{1}{\Gamma})sin\phi, cos(\theta_0+\frac{1}{\Gamma})]$, the outgoing photon vector, $\vec{k}_2$ is parallel to $\hat{n}$. The velocity vectors of the outflow, which form a cone around the z axis, are defined by $\hat{\beta} = [sin\theta_0 cos\phi, sin\theta_0 sin\phi, cos\theta_0]$.

In the fully general 3-D scattering problem, the degree of polarization depends on the scattering angle w', where $cosw' = \hat{k}_1' \cdot \hat{k}_2'$, whereas the Doppler boosting of the intensity of the scattered radiation depends on $\theta \equiv arcos(\hat{\beta}\cdot\mathbf{\hat{n}})$ as $I(\nu) = I'(\nu')[\Gamma(1-\beta cos\theta)]^{-k}$. Here $k = 3-\alpha$ where α is the spectral index. The difficulty is that w' and θ' are defined relative to different axes. However, in the case that the incoming photon vector makes an angle of $cos^{-1}\beta$ with the velocity vector, the frequency in the frame of the scatterer is given by

$$\nu' = \frac{\nu_1}{\Gamma}, \tag{8}$$

and we assume that ν' is conserved in the scattering. We can use the invariance of $k_{1\mu}k_2^{\mu}$ to write

$$1 - cosw' = \Gamma^2\frac{\nu_2}{\nu_1}(1-cosw) = \Gamma^2(1+\beta cos\theta')(1-cosw). \tag{9}$$

Using the aberration formula,

$$1+\beta cos\theta' = 1+\beta\frac{(cos\theta-\beta)}{1-\beta cos\theta} = \frac{1}{\Gamma^2(1-\beta cos\theta)} \tag{10}$$

we can write cosw' in terms of laboratory coordinates as

$$cosw' = 1 - \frac{(1-cosw)}{(1-\beta cos\theta)} = \frac{(cosw - \beta cos\theta)}{(1-\beta cos\theta)}. \tag{11}$$

The Doppler boost and polarization expressions from emission from a thin cone of opening angle θ_o can now be expressed as functions of ϕ, Γ and θ_o using the basic expressions

$$cosw = sin(\theta_0+\Delta)sin(\theta_0+\frac{1}{\Gamma})cos\phi + cos(\theta_0+\Delta)cos(\theta_0+\frac{1}{\Gamma}) \tag{12}$$

and

$$\beta cos\theta = \vec{\beta}\cdot\hat{n} = \beta[sin\theta_0 sin(\theta_0+\Delta)cos\phi + cos\theta_0 cos(\theta_0+\Delta)] \tag{13}$$

In the limit of $1/\Gamma \ll \theta_o$, and $\Delta \ll \theta_o$, we can expand the trigonometric formulae and obtain

$$1 - cosw \sim [(1/\Gamma - \Delta)^2 + \tilde{\phi}^2]/2, \tag{14}$$

where $\tilde{\phi} = sin\theta_o\phi$, and

$$1 - \beta cos\theta \sim (1/\Gamma^2 + \Delta^2 + \tilde{\phi}^2)/2 \tag{15}$$

whence

$$cosw - \beta cos\theta \sim \Delta/\Gamma \tag{16}$$

Thus, in the viewing direction $\Delta = 0$, $cosw' = 0$ and the radiation scattered into this direction after a single scattering is nearly *fully polarized*. It can be further shown in the single scattering limit that when $\Gamma \gg 1$, the polarization vectors of all the rays scattered into a line of sight with $\Delta = 0$ have polarization nearly perpendicular to the cone axis as seen projected on the sky, so that their sum is nearly fully polarized as well.

In the limit $1/\Gamma \ll \theta_o$ and $\Delta \ll \theta_o$, the beaming direction $\Delta = 0$ corresponds to maximum polarization. The direction of maximum total intensity (i.e. the sum of intensities of both polarizations) does not coincide with the beaming direction, but even if we assume a detection threshold for a gamma ray polarimeter that is a function of the total intensity, the relative probability of detecting high polarization is considerably higher than in the "head-on" approximation. Writing the total intensity in the above limits, where relevant ϕ are at $\phi \ll 1$, as

$$I(\Delta) \propto \int_{-\pi}^{\pi} (1 - \beta\cos\theta)^{-k}(1 + cosw'^2)sin\theta_o d\phi \sim \int_{-\infty}^{\infty} (1 - \beta\cos\theta)^{-k}(1 + cosw'^2)d\tilde{\phi}, \tag{17}$$

we find, performing the integrals, that

$$I(\Delta) \propto [\Gamma^{-2} + \Delta^2]^{k-1/2}\left(1 + \frac{4(2k+1)(2k-1)(\Delta\Gamma)^2}{(2k+2)(2k)[1 + (\Delta\Gamma)^2]^2}\right) \tag{18}$$

For k=3,

$$\frac{I(\Delta = 0)}{I(\Delta = 1/\Gamma)} = 2^{5/2}\frac{48}{83} \sim 3.3, \tag{19}$$

and, in a Euclidean space,

$$\frac{V_{max}(\Delta = 0)}{V_{max}(\Delta = 1/\Gamma)} \sim 6. \tag{20}$$

To summarize, scattering material in the Compton sailing state beams the polarized scattered component forward, and, for a roughly homogenous source distribution, a nearly fully polarized beam has the highest V_{max}. The less intense unpolarized component, which by invariance of emitted power should be about as strong when averaged over a solid angle, presumably covers a larger solid angle, so for a source distribution with a low $\frac{V}{V_{max}}$, we might see mostly weakly-polarized GRB's. The above calculation has several simplifying assumptions: it is for the analytically tractable case $\theta_o\Gamma \gg 1$, $\Delta\theta \ll \theta_o$ i.e. scattering off a hollow cone into a thin annulus. It neglects penetration to finite optical depths, multiple scattering, finite source size, and viscosity within the sheath. Many of these issues will be addressed in a subsequent paper with detailed numerical calculations (Levinson & Eichler astro-ph/0402457).

Finally, it is worth recalling that the bulk Lorentz factor Γ of GRB fireballs, while usually estimated from observational constraints to be several hundred, has never been calculated from first principles. The phenomenon of Compton sailing suggests that a) there may be a connection between Γ and the geometry of the flow and b) Γ may vary continuously between the fireball and the ejected, supernova-associated material from the envelope of the GRB's host star. It suggests the possibility of calculating Γ theoretically. While such a calculation would be formidable, the problem seems sufficiently well posed that Γ is no longer a mere free parameter imposed on the initial conditions of the fireball.

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